Black rings from fluid mechanics

Subhaneil Lahiri

based on arXiv:0705.3404 [hep-th] with Shiraz Minwalla
and arXiv:0903.4734 [hep-th] with Jyotirmoy Bhattacharya

May 1, 2009

Higher dimensional gravity

General relativity makes sense in any # of dimensions.

D is a parameter.

We vary parameters to understand the theory better: c.f. coupling constants, gauge groups, . . .

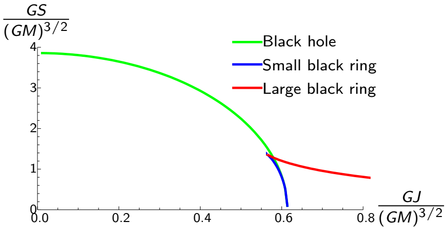
Sometimes we need extra dimensions: string theory, large extra dimensions scenarios, . . .

Is gravity the same in $D > 4$?

Black holes in four vs. five dimensions

In four dimensions there are horizon topology and black hole uniqueness theorems.

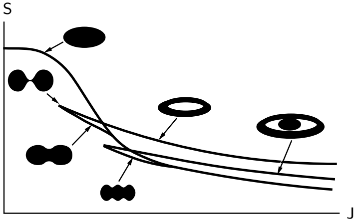
In five dimensions, we are allowed an $S^1 \times S^2$ horizon as well – the black ring. [Emparan, Reall]



For a range of energies and angular momenta, it is possible to have two black ring and one black hole solutions - violating uniqueness.

Higher dimensions

For $D \geq 6$: no exact solutions (except Myers-Perry). Approximate solutions for $R_{S^1} \gg R_{S^3}$.



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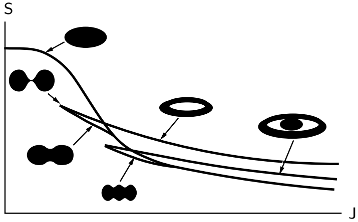

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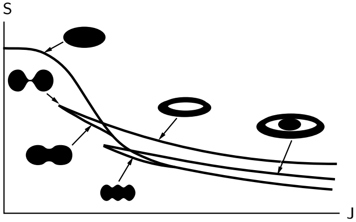

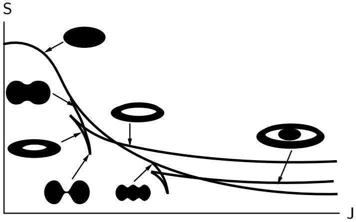

[Emparan et al.]

Other topologies? [Galloway, Schön]

The AdS/CFT correspondence

Gravitational theory	$\Leftrightarrow$	Non-gravitational theory
Low curvature		Strong coupling
Black holes		Deconfinement

Black holes and fluid mechanics

At long wavelengths, deconfined plasma described by fluid mechanics.

Only input: equation of state, transport coefficients. Also works at strong coupling.

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Equation of state: black hole thermodynamics, static case.
Transport coefficients: small fluctuations, e.g. $\eta/s = 1/4\pi$. [Son,Starinets]

$\implies$ universal features of black holes at long wavelengths.

Outline

1. Motivation
2. Plasmaball setup
3. Relativistic fluid mechanics
4. Three dimensional configurations
5. Higher dimensional generalisations
6. Summary

Plasmaballs in confining theories

Plasmaballs are a bubbles of deconfined phase, surrounded by confined phase, held together by surface tension.

Focus on theories that come from compactifying conformal theories on a Scherk-Schwarz circle.

Leads to confining theory.

Confined phase

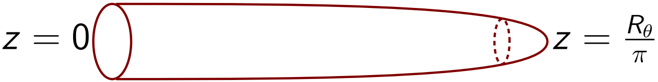
At low temperatures, gravity dual: AdS soliton:

$$\mathrm{d}s^2 = \frac{R_{\mathrm{AdS}}^2}{z^2}\left(-\mathrm{d}t^2 + F_{R_\theta}(z)\,\mathrm{d}\theta^2 + \mathrm{d}\vec{x}^2 + \frac{1}{F_{R_\theta}(z)}\,\mathrm{d}z^2\right),$$

where $F_a(u) = 1 - \left(\frac{\pi z}{a}\right)^4$ and $R_{\mathrm{AdS}}^2 = \sqrt{\lambda}\alpha'$. [Witten]

Small z: Poincaré AdS_5 with one compact direction.

At $z = R_\theta/\pi$, the θ circle contracts: space stops.



Deconfined phase

At high temperatures: the black brane:

$$\mathrm{d}s^2 = \frac{R_{\mathrm{AdS}}^2}{z^2} \left(-F_\beta(z)\,\mathrm{d}t^2 + \mathrm{d}\theta^2 + \mathrm{d}\vec{x}^2 + \frac{1}{F_\beta(z)}\,\mathrm{d}z^2 \right).$$

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Horizon at $z = \frac{\beta}{\pi}$. Temperature: $\mathcal{T} = 1/\beta$.

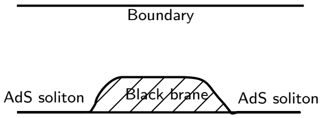
Dominant phase above transition temperature, $\mathcal{T}_\mathrm{c} = \frac{1}{R_\theta}$.

The equation of state of the dual plasma can be found from this gravity solution.

$$\mathcal{P} = \frac{\alpha}{\mathcal{T}_\mathrm{c}}\left(\mathcal{T}^4 - \mathcal{T}_\mathrm{c}{}^4\right).$$

Plasmaball solutions

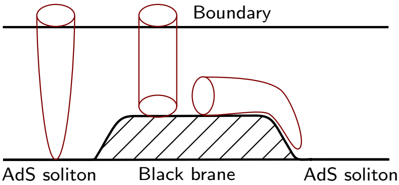
On the bulk side, deep interior looks like black brane. Far from the plasmaball, it looks like the AdS soliton. There is a domain wall that interpolates between the two.



In the limit of infinitely large radius, a numerical domain wall solution has been found. The surface tension and thickness can be computed from this solution. [Aharony, Minwalla, Wiseman]

Topology

The Scherk-Schwarz circle does not contract in the black brane region but does contract in the AdS soliton region.



Horizon topology: fibre circle over the plasmaball, contracting at surfaces.

Fluid mechanics

The equations of motion are $\nabla_\mu T^{\mu\nu} = 0$. The dynamical input is in specifying $T^{\mu\nu}$.

For long wavelengths, we need only go up to one derivative terms:
$T^{\mu\nu} = T^{\mu\nu}_{\text{perfect}} + T^{\mu\nu}_{\text{dissipative}}$.

Coefficients depend on $\mathcal{T}$. Determined from static black brane.

This approximation breaks down at surfaces – but at scales $\gg$ surface thickness we can replace these regions with a δ-function localised surface tension.

Three dimensional configurations

Rigid rotation: $(u^t, u^r, u^\phi) = \gamma(1, 0, \Omega)$, where $\gamma = \frac{1}{\sqrt{1-v^2}}$.
Centripetal force provided by pressure gradient.

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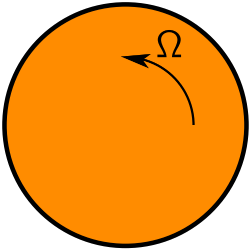
Surfaces: $\mathcal{P} = \pm\frac{\sigma}{r}$. Relates $(\mathcal{T}/\gamma)$ to Ω and position of surface.

Solutions

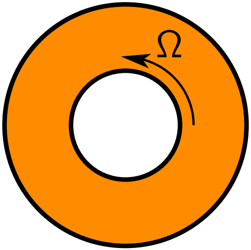
We find two types of solution:

Plasmaballs



B^2

Plasmarings



$S^1 \times B^1$

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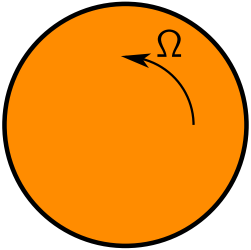
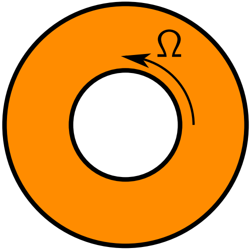


$$\begin{array}{ccc} S^1 & \longrightarrow & S^3 \\ & & \downarrow \\ & & B^2 \end{array}$$

Plasmarings



$$\begin{array}{ccc} S^1 & \longrightarrow & S^1 \times S^2 \\ & & \downarrow \\ & & S^1 \times B^1 \end{array}$$

Thermodynamics

We compute the thermodynamic properties of the whole solution with

$$E = \int \mathrm{d}^2 x \left(T^{tt} \right) ,$$

$$L = \int \mathrm{d}^2 x \left(r^2 T^{t\phi} \right) ,$$

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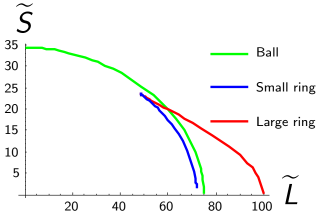
Then we compute an overall temperature and angular velocity via

$$\mathrm{d}E = T \mathrm{d}S + \Omega \mathrm{d}L,$$

we find

$$T = \frac{\mathcal{T}}{\gamma}, \qquad \Omega \text{ as before}.$$

Phase diagram



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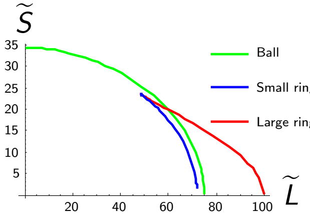
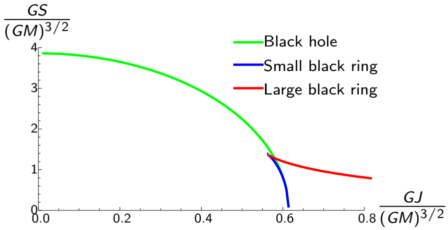




Topologies in six dimensions

S^4

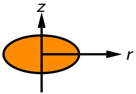
$S^3 \times S^1$

$S^2 \times S^2$

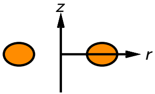
$S^2 \times T^2$

Topologies in six dimensions

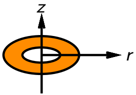
$$S^4 \downarrow B^3$$



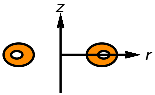
$$S^3 \times S^1 \downarrow B^2 \times S^1$$



$$S^2 \times S^2 \downarrow B^1 \times S^2$$



$$S^2 \times T^2 \downarrow B^1 \times T^2$$



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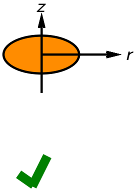
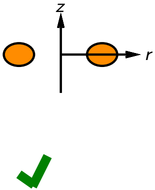


$$S^3 \times S^1 \downarrow B^2 \times S^1$$



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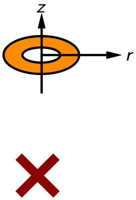

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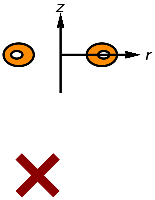


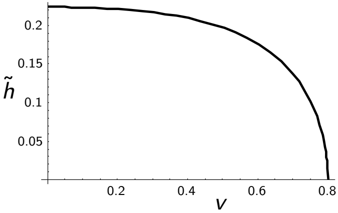
Solving equations of motion

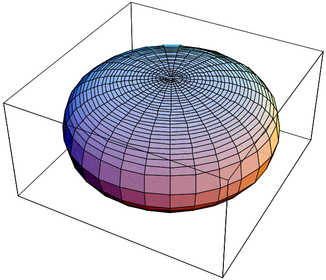
Again: rigid rotation $(u^t, u^r, u^\phi, u^z) = \gamma(1, 0, \Omega, 0)$.

Again: $\frac{\mathcal{T}}{\gamma} = T =$ constant.

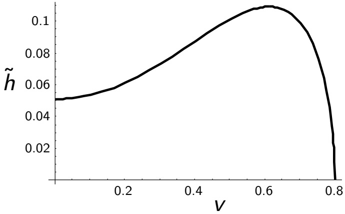
Now: surface satisfies $\mathcal{P} = \sigma K^\mu_\mu$.

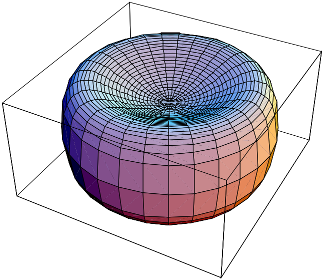
Ordinary balls



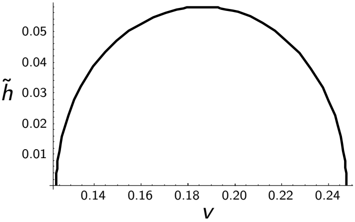


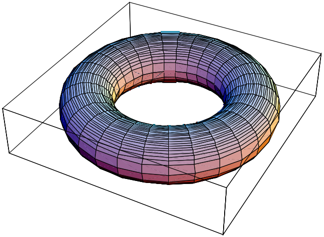
Pinched balls



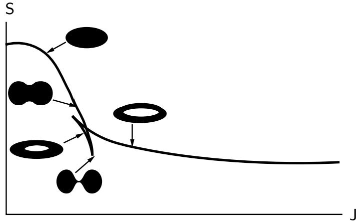


Rings





Phase diagram



[Bhardwaj,Bhattacharya]

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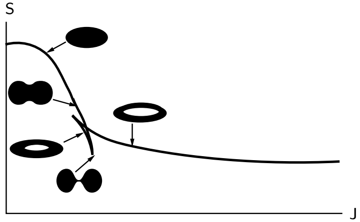

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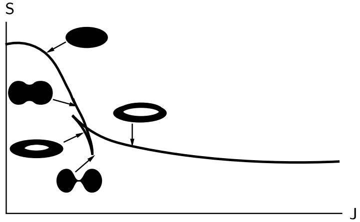
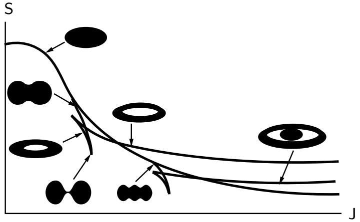




[Bhardwaj,Bhattacharya]

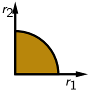
Topologies in seven dimensions

S^5 $\quad$ $S^4 \times S^1$ $\quad$ $S^3 \times T^2$

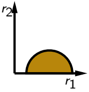
$S^3 \times S^2$ $\quad$ $S^2 \times S^2 \times S^1$ $\quad$ $S^2 \times T^3$

Topologies in seven dimensions

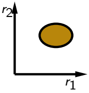
$$\begin{array}{c} S^5 \\ \downarrow \\ B^4 \end{array}$$



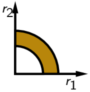
$$\begin{array}{c} S^4 \times S^1 \\ \downarrow \\ B^3 \times S^1 \end{array}$$



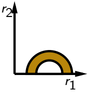
$$\begin{array}{c} S^3 \times T^2 \\ \downarrow \\ B^2 \times T^2 \end{array}$$



$$\begin{array}{c} S^3 \times S^2 \\ \downarrow \\ B^1 \times S^3 \end{array}$$



$$\begin{array}{c} S^2 \times S^2 \times S^1 \\ \downarrow \\ B^1 \times S^2 \times S^1 \end{array}$$



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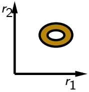


Approximate solutions

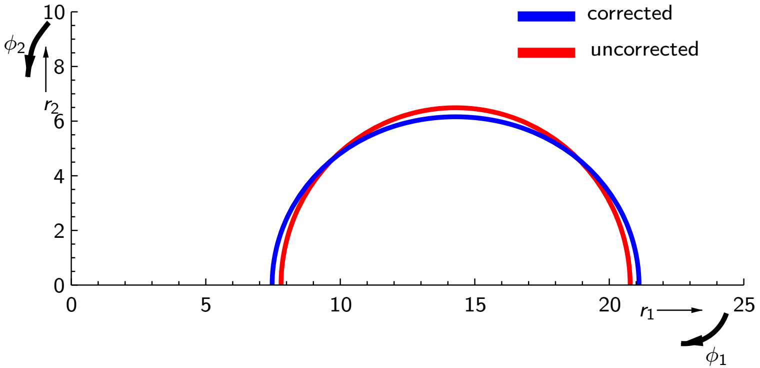
For ring, $B^3 \times S^1$, take $\epsilon = \frac{R_{B^3}}{R_{S^1}}$ small.

For 'torus', $B^2 \times T^2$, take $\epsilon = \frac{R_{B^2}}{R_{T^2}}$ small.

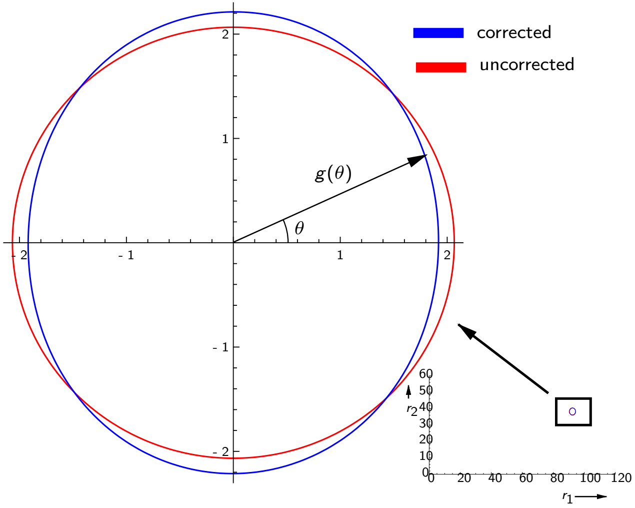
Expand in ϵ. At $\mathcal{O}(\epsilon^0)$ – just a tube.

Similar to black-fold construction of Emparan et al.

Ring



Torus


corrected
uncorrected
$g(\theta)$
θ
2
1
- 1
- 2
- 2
- 1
1
2
60
50
40
30
20
10
0
0
20
40
60
80
100
120
r_2
r_1

Summary

We can get insight to some problems in classical gravity from fluid mechanics in AdS/CFT.

In five dimensions – qualitative agreement with flat space gravity.

In six dimensions – proposal for phase diagram.

In seven dimensions – new topology.

Future: numerical solutions for $D = 7$, phase diagram.

Gregory-Laflamme vs. Plateau-Rayleigh. [Caldarelli et al.]